Measurement and Calibration of a Universal Six-Port Network Analyzer

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***Abstract*—A general theory of measuring multiport networks is presented in this paper. In order to measure microwave multiport networks conveniently a method involving the stepwise reduction of the order of the network under test is suggested. All formulas for calibrating and measuring the triple six-port network analyzer (TSPNA), which is a universal six-port network analyzer, are given without any ambiguity. The procedures for calibration and measurement are very simple. No standard three-port network is needed to calibrate the six-port system. Finally, the error caused by nonideal isolation is discussed.**

I. Introduction

SINCE THE six-port theory presented by Hoer and Engen [1], [2], in 1972, microwave measurement theory and techniques have made great progress. With the development of computer-aided measurement, microwave measurement techniques have become more accurate. A dual six-port network analyzer (DSPNA) was proposed by Hoer [3] in 1977. Since then many scholars [4]–[6] have been engaged in the calibration of the DSPNA and the measurement of two-port networks, and the calibration of the DSPNA has been greatly improved.

In the field of microwave techniques a great variety of microwave multiport networks are used. How to measure all the scattering parameters of a multiport network is a problem encountered in our research.

The single-port network can be measured with a single six-port network analyzer (SSPNA), and a two-port network with a DSPNA. Can we measure a multiport network with a multi-six-port network analyzer (MSPNA)? Further, is there a universal six-port network analyzer (USPNA) which can be conveniently used to measure a variety of microwave networks? Li [7] has suggested an isolated N-six-port network analyzer (INSPNA). Speciale has developed a method for determining the $n \times n$ complex scattering matrix of a multiport network in a single measurement [8]. This method requires an unconventional type of multiport network analyzer. A more conventional method of performing such measurements on a two-port automated network analyzer requires perfecting matched loads of $(n-2)$. This requirement cannot be met in practice with sufficient accuracy. A rigorous technique for measuring the scattering matrix of a multiport network with a two-port network analyzer was described in [9]. The key to calibrating measurements of the scattering matrix of multiport networks with a two-port network analyzer is the renormalization transforms of the scattering parameters. The renormalization transforms were originally derived by Woods for networks with up to six ports [10]–[12]. A generalized form of the renormalization transform was given in [13]. Dropkin simplified the transform and pointed out that in some cases the inverse required by the transform may not even exist [14].

In this paper a general theory of measuring multiport networks is presented and a method of reducing the order of the network under test step by step (ROM) is suggested for measuring microwave multiport networks conveniently.

All associated equations are linearized without any ambiguity. The calibration and measurement of the triple six-port network analyzer (TSPNA) are discussed in detail, and it is shown that the TSPNA is a USPNA. Using the ROM method with the TSPNA one can measure all the scattering parameters of any multiport network, thereby avoiding inverse operations and saving connection time when $n = 3, 4$.

It is assumed that all the isolators used in the DSPNA and TSPNA discussed in the following (see Figs. 2 and 3) are ideal ones. Nonideal isolators used in the six-port system will introduce errors. The isolation property of a nonideal isolator can be described by an isolation factor $I = S_{12}S_{21}/S_{11}$. The relationship between the error factor and the isolation factor is given.

II. General Theory

An N-port network can be characterized by an $n \times n$ scattering matrix, that is,

$$b = Sa \tag{1}$$

where b is a reflected wave column matrix, a is an incident wave column matrix, and S is an $n \times n$ scattering matrix. Assuming that a load with a reflection coefficient of k is connected to the kth port of the N-port network, the N-port network and the load become an $(N-1)$-port network$^{(k)}$, the superscript k denoting the elimination of the kth port of the N-port network. By substituting the relation

$$a_k = \Gamma_k b_k \tag{2}$$

into (1), the $n \times n$ matrix S is reduced to an $(n-1) \times (n-1)$ matrix $S^{(k)}$. The matrix $S^{(k)}$ is obtained by eliminating the kth row and kth column of the matrix S and substituting the element $S_{ij}^{(k)}$ for the element S_{ij} of the

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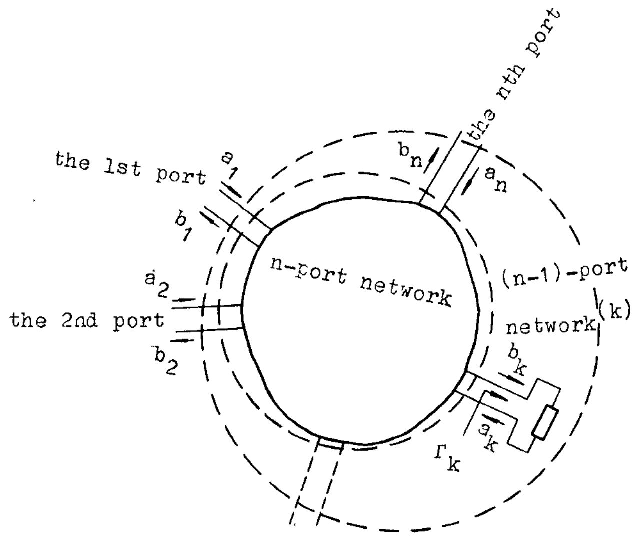

Fig. 1. An N-port network becomes an $(N-1)$-port network by eliminating the kth port of the N-port network.

matrix S. The relationship between $S_{ij}^{(k)}$ and S_{ij} is

$$S_{ij}^{(k)} = S_{ij} + \frac{\Gamma_k S_{ik} S_{kj}}{1 - S_{kk}\Gamma_k}, \qquad i,\ j,\ k = 1,2,\cdots,n \qquad i,\ j \neq k. \tag{3}$$

In order to affirm the relationships between elements of the two matrices S and $S^{(k)}$, the subscripts of $S_{ij}^{(k)}$ are not changed.

An N-port network can construct n different $(N-1)$-port networks by eliminating one of the n ports of the network by means of the method described above. For a single k, (3) denotes $(n-1)^2$ $S_{ij}^{(k)}s$; when k equals $1,2,\cdots,n$ successively, (3) denotes $n(n-1)^2 S_{ij}^{(k)}s$. If all scattering parameters of $(N-1)$-port networks formed by the N-port network are found, all scattering parameters of the N-port network can be obtained from (3) in principle. (It will be seen that the scattering parameters of the N-port network cannot be found by the ROM method when $n \leqslant 3$.)

The first step is to solve for diagonal elements of the matrix S. Connecting a load with a reflection coefficient of Γ_k at the kth port of the N-port network, thus eliminating the kth port, we make the N-port network become an $(N-1)$-port network$^{(k)}$ (Fig 1). At the ith port, $i \neq k$, (3) becomes

$$S_{ii}^{(k)} = S_{ii} + \frac{\Gamma_k S_{ik} S_{ki}}{1 - S_{kk}\Gamma_k}, \qquad i \neq k. \tag{4}$$

Similarly, eliminating the ith port we obtain an $(N-1)$-port network$^{(i)}$. And at the kth port,

$$S_{kk}^{(i)} = S_{kk} + \frac{\Gamma_i S_{ki} S_{ik}}{1 - S_{ii}\Gamma_i}, \qquad k \neq i. \tag{5}$$

From (4) and (5), $S_{ik}S_{ki}$ is found to be

$$\begin{aligned} S_{ik}S_{ki} &= \left(S_{ii}^{(k)} - S_{ii}\right)(1 - S_{kk}\Gamma_k)/\Gamma_k \\ &= \left(S_{kk}^{(i)} - S_{kk}\right)(1 - S_{ii}\Gamma_i)/\Gamma_i. \end{aligned} \tag{6}$$

From the last equation of (6) we obtain

$$\left(1 - \Gamma_i S_{ii}^{(k)}\right)\Gamma_k S_{kk} - \left(1 - \Gamma_k S_{kk}^{(i)}\right)\Gamma_i S_{ii} = \Gamma_k S_{kk}^{(i)} - \Gamma_i S_{ii}^{(k)}, \qquad i,\ k = 1,2,\cdots,n; \quad i \neq k \tag{7}$$

where Γ_k, Γ_i are reflection coefficients of two known loads. $S_{ii}^{(k)}$ and $S_{kk}^{(i)}$ are the diagonal scattering parameters of both the $(N-1)$-port network$^{(k)}$ and the network$^{(i)}$, respectively. Because $S_{kk}^{(i)}$ and $S_{ii}^{(k)}$ are known by measurement, (7) represents a set of linear equations of the diagonal elements of the scattering matrix S of the N-port network. The number of equations in (7) is only $\frac{1}{2}n(n-1)$, when i and k vary from 1 to n, respectively. The number of diagonal elements in the N-port network is n when $\frac{1}{2}n(n-1) \geqslant n$; that is, when $n \geqslant 3m$ all diagonal elements of the matrix S can be found from (7).

When $n > 3$ the number of equations in (7) is more than that of the diagonal elements. The surplus equations are not independent. For the sake of convenience, making k and i equal 1 and 2, 2 and 3, $\cdots$, n and 1, respectively and successively, we obtain n equations altogether. Writing the equations in matrix form, we have

$$A_d S_r = B_d \tag{8a}$$

$$S_r = (\Gamma_1 S_{11}, \Gamma_2 S_{22}, \cdots, \Gamma_n S_{nn})^t \tag{8b}$$

$$B_d = \begin{bmatrix} \Gamma_1 S_{11}^{(2)} - \Gamma_2 S_{22}^{(1)} \\ \Gamma_2 S_{22}^{(3)} - \Gamma_3 S_{33}^{(2)} \\ \vdots \\ \Gamma_n S_{nn}^{(1)} - \Gamma_1 S_{11}^{(n)} \end{bmatrix} \tag{8c}$$

$$A_d = \begin{bmatrix} 1 - \Gamma_2 S_{22}^{(1)} & -1 + \Gamma_1 S_{11}^{(2)} & 0 & \cdots & 0 \\ 0 & 1 - \Gamma_3 S_{33}^{(2)} & -1 + \Gamma_2 S_{22}^{(3)} & \cdots & 0 \\ \cdots & & & & \\ 0 & 0 & 0 & \cdots & -1 + \Gamma_{n-1} S_{n-1,n-1}^{(n)} \\ -1 + \Gamma_n S_{nn}^{(1)} & 0 & 0 & \cdots & 1 - \Gamma_1 S_{11}^{(n)} \end{bmatrix} \tag{8d}$$

TABLE I

Number of ports	2	3	4	5	6	n	NOTE
Number of whole equations	2	12	36	80	150	$n(n-1)^2$	Eq. (3)
Number of S_{kk}	2	3	4	5	6	n	Diagonal elements
Number of equations for S_{kk}	1*	3	6	10	15	$\frac{1}{2}n(n-1)$	Eq. (7)
Number of S_{ij}, $i \neq j$	2	6	12	20	30	$n(n-1)$	Nondiagonal elements
Number of equations for S_{ij}	0*	3*	12	30	60	$\frac{1}{2}n(n-1)(n-2)$	Eq. (10)

The asterisk indicates that the number of equations is less than that of the unknown S_{ij}.

Equation (8) comprises a set of linear equations of the diagonal elements. Using Cramer's rule S_{kk} is found to be

$$S_{kk} = \frac{1}{\Gamma_k} \sum_{j=1}^{n} B_{dj} A_{djk} / \det A_d, \qquad k = 1, 2, \cdots, n \quad (8e)$$

where t denotes the transpose operation, B_{dj} is an element of the column matrix B_d, and A_{djk} is an algebraic complementary minor of the element of the jth row and the kth column of the matrix A_d.

The second step is to solve for the nondiagonal elements of the matrix S. Substituting (8e) into (3) we obtain a product, $S_{ik}S_{ki}$, of a couple of nondiagonal elements. The number of all products is $\frac{1}{2}n(n-1)$. We have used $n(n-1)$ equations in solving (8) for the diagonal elements of the matrix S, so there are $n(n-1)(n-2)$ equations remaining on nondiagonal elements. Using (3) we can transform the $n(n-1)(n-2)$ equations into linear ones.

From (3) we have

$$\Gamma_k S_{ik} S_{kj} = \left(S_{ij}^{(k)} - S_{ij}\right)(1 - \Gamma_k S_{kk}), \qquad k \neq i, j. \quad (9)$$

By retaining the constant k and exchanging subscripts i and j of the scattering parameters in (9), we get another equation (see the Appendix). Multiplying both sides of the equation obtained by (9), respectively, we have

$$S_{ji}^{(k)} S_{ij} + S_{ij}^{(k)} S_{ji} = S_{ij}^{(k)} S_{ji}^{(k)} + S_{ij} S_{ji} - \left(S_{ii}^{(k)} - S_{ii}\right)\left(S_{jj}^{(k)} - S_{jj}\right),$$
$$i, j = 1, 2, \cdots, n; \quad i \neq j; \quad i, j \neq k. \quad (10)$$

Equation (10) represents a set of linear equations of the nondiagonal elements of the matrix S. The matrix S has $n(n-1)$ nondiagonal elements. The number of equations given by (10) is $\frac{1}{2}n(n-1)(n-2)$, when $\frac{1}{2}n(n-1)(n-2) \geqslant n(n-1)$; that is, when $n \geqslant 4$, all the nondiagonal elements of the matrix S can be found.

Since (10) is symmetrical, S_{ij} and S_{ji} can be found in couplet. Substituting h in (10) for k, we have

$$S_{ji}^{(h)} S_{ij} + S_{ij}^{(h)} S_{ji} = S_{ij}^{(h)} S_{ji}^{(h)} + S_{ij} S_{ji} - \left(S_{ii}^{(h)} - S_{ii}\right)\left(S_{jj}^{(h)} - S_{jj}\right),$$
$$i, j = 1, 2, \cdots, n; \; i \neq j; \; i, j \neq h. \quad (11)$$

When $h \neq k$, from (10) and (11) S_{ij} and S_{ji} are found to be

$$S_{ij} = \frac{\begin{vmatrix} E_{ij}^{(k)} & S_{ij}^{(k)} \\ E_{ij}^{(h)} & S_{ij}^{(h)} \end{vmatrix}}{\begin{vmatrix} S_{ji}^{(k)} & S_{ij}^{(k)} \\ S_{ji}^{(h)} & S_{ij}^{(h)} \end{vmatrix}} \quad (12a)$$

$$S_{ji} = \frac{\begin{vmatrix} S_{ji}^{(k)} & E_{ij}^{(k)} \\ S_{ji}^{(h)} & E_{ij}^{(h)} \end{vmatrix}}{\begin{vmatrix} S_{ji}^{(k)} & S_{ij}^{(k)} \\ S_{ji}^{(h)} & S_{ij}^{(h)} \end{vmatrix}} \quad (12b)$$

$$E_{ij}^{(k)} = S_{ij}^{(k)} S_{ji}^{(k)} - S_{ij} S_{ji} - \left(S_{ii}^{(k)} - S_{ii}\right)\left(S_{jj}^{(k)} - S_{jj}\right) \quad (12c)$$

$$E_{ij}^{(h)} = S_{ij}^{(h)} S_{ji}^{(h)} - S_{ij} S_{ji} - \left(S_{ii}^{(h)} - S_{ii}\right)\left(S_{jj}^{(h)} - S_{jj}\right),$$
$$i, j = 1, 2, \cdots, n; \quad i \neq j; \quad i, j \neq k \neq h. \quad (12d)$$

For convenient calculation, we usually select $i < j$, $h = k + 1 = j + 2$. When $i \geqslant 3$, $j \geqslant n-1$, $k - n$ substitutes for k, and $h - n$ for h. When determining $S_{1,n-1}$ and $S_{n-1,1}$ we select $k = n$, $h = 2$; for $S_{1,n}$ and $S_{n,1}$, select $k = 2$ and $h = 3$; for $S_{2,n-1}$ and $S_{n-1,2}$, select $k = n$ and $h = 1$; for $S_{2,n}$ and $S_{n,2}$, select $k = 1$ and $h = 3$. Thus S_{12} and S_{21}, $S_{13}, \cdots, S_{1n}$ and S_{n1}, S_{23} and S_{32}, S_{ij} and S_{ji} through $S_{n-1,n}$ and $S_{n,n-1}$ can be found successively.

In the process of solving the diagonal and nondiagonal elements of the matrix S, we can see that there is not always a definite solution for any integer n. The relationships between the number of network ports, the number of unknown S_{ij}, and the number of equations on S_{ij} are given in Table I. From this table we can see that the number of linear equations of diagonal elements is $\frac{1}{2}n(n-1)$, and the number of linear equations of nondiagonal elements is $\frac{1}{2}n(n-1)(n-2)$; altogether they are $\frac{1}{2}n(n-1)^2$. When $\frac{1}{2}n(n-1)^2 \geqslant n^2$, all elements of the matrix S can be found by the ROM method, and there is no ambiguity. For example, when $n = 4$ the four-port network will be reduced to three triple-port networks to be measured, provided all scattering parameters of the three triple-port networks have been found. The scattering pa-

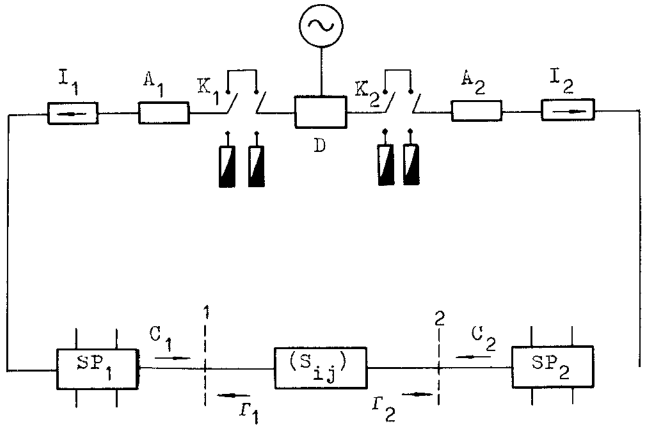

Fig. 2. A scheme of DSPNA.

rameters of the four-port network will be obtained from (7) and (10).

Multiport networks with more than four ports can be measured by reducing them step by step to three-port networks. From Table I, however, we can see that the number of equation for S_{kk} is 1 when $n=2$, but the number of unknown S_{kk}'s is 2; the number of equations for S_{ij} is 0, but the number of unknown S_{ij}'s is 2; the number of equations for S_{kk} is the same as the unknown S_{kk}'s when $n=3$, but the number of equations for S_{ij} $(i \neq j)$ is just half that of the unknown S_{ij}'s. So two-port and three port networks cannot be measured by the ROM method. The measurements of two-port networks and three-port networks will be discussed below.

III. Calibration and Measurement of DSPNA

A scheme of the DSPNA is shown in Fig. 2. The DSPNA is composed of a divider, switches, attenuators, isolators, matching loads, and six-port networks. The two-port network under test with scattering parameters S_{ij}, $i, j=1,2$, is connected at reference planes 1 and 2. Γ_1 and Γ_2 are the reflection coefficients seen in the six-ports sp_1 and sp_2 from reference planes 1 and 2, respectively, and c_1 and c_2 are the incident waves coming into the two-port network under test from the source. Γ_1, Γ_2 and the ratio of c_2 to c_1 are system constants.

Here sp_1 and sp_2 are two six-port networks being calibrated. For the method of calibrating six-port networks, see [15]. Here we shall discuss only the calibration of the DSPNA system.

The signal coming from the source is divided into two parts and comes into the right and left branches of the DSPNA, respectively.

The two branches are symmetrical in structure. The attenuators are used for limiting the power so as not to exceed the dynamic range of the power meters of the six-ports. The isolators are ideal, which guarantees that (i) regardless of the state the switch K_i is in, the reflection coefficient Γ_i seen in the six-port sp_i from the reference plane i will always be a constant and (ii) regardless of which unknown two-port network is connected at reference planes 1 and 2, the ratio of c_2 to c_1 will always be a constant [16]. The following equations are derived following the method in the literature [16].

When switch K_1 connects with attenuator A_1 (making the six-port sp_1 connect directly with the source, called switch K_1 closed), whereas switch K_2 connects with the matching load (making sp_2 open with the source, called K_2 open), the reflection coefficient Γ_{1p} which is measured by sp_1 is

$$\Gamma_{1p} = (S_{11} - \Gamma_2\Delta)/(1 - \Gamma_2 S_{22})$$
$$\Delta = S_{11}S_{22} - S_{12}S_{21}. \tag{13}$$

When K_1 is open and K_2 closed, the reflection coefficient Γ_{2p} measured by sp_2 is

$$\Gamma_{2p} = (S_{22} - \Gamma_1\Delta)/(1 - \Gamma_1 S_{11}). \tag{14}$$

When K_1 and K_2 are both closed, the reflection coefficients measured by the two six-ports are denoted by the subscript a, e.g. Γ_{1a} and Γ_{2a}. From the superposition theorem they are

$$\Gamma_{1a} = \frac{S_{11} - \Gamma_2\Delta + S_{12}c_2/c_1}{1 + \Gamma_1 S_{12}c_2/c_1 - \Gamma_2 S_{22}} \tag{15}$$

$$\Gamma_{2a} = \frac{S_{22} - \Gamma_1\Delta + S_{21}c_1/c_2}{1 + \Gamma_2 S_{21}c_1/c_2 - \Gamma_1 S_{11}}. \tag{16}$$

Using (13)–(16), the system constants of the DSPNA can be calibrated. The calibration steps are as follows.

First, the measurement port of sp_1 connects directly with that of sp_2, that is, a transmission line of zero length (or a line with arbitrary length, thus the formulas will be slightly different), as a standard two-port network is connected at reference planes 1 and 2. When K_1 is closed and K_2 open, Γ_{1p}' is measured by sp_1 from (13), that is,

$$\Gamma_{1p}' = \Gamma_2. \tag{17}$$

Then, when K_1 is open and K_2 closed, Γ_{2p}' is measured by sp_2 from (14), that is,

$$\Gamma_{2p}' = \Gamma_1. \tag{18}$$

Finally, when K_1 and K_2 are both closed, Γ_{1a}' is measured by sp_1; from (15) the ratio of c_2 to c_1 is obtained (there are eight power readings, but only four of them are used):

$$c_2/c_1 = (\Gamma_{1a}' - \Gamma_2)/(1 - \Gamma_1\Gamma_{1a}'). \tag{19}$$

Thus under three switch states, the three system constants Γ_1, Γ_2, and c_2/c_1 are obtained from 16 power readings.

Next, connect the two ports of the unknown two-port network with measurement ports of sp_1 and sp_2, respectively. The measurement steps are the same as those of calibrating a DSPNA:

(i) when K_1 is closed and K_2 open, Γ_{1p} is measured by sp_1;
(ii) when K_1 is open and K_2 closed, Γ_{2p} is measured by sp_2;
(iii) when K_1 and K_2 are both closed, Γ_{1a} and Γ_{2a} are measured by sp_1 and sp_2, respectively.

By substituting Γ_{1p}, Γ_{2p}, Γ_{1a}, and Γ_{2a} into (13)–(16), all scattering parameters of the two-port network under test can be determined:

$$S_{11} = (\Gamma_{1p} - \Gamma_2\beta_1\beta_2)/\beta_3 \tag{20a}$$

$$S_{12} = \beta_1(1-\Gamma_2\Gamma_{2p})/\beta_3 \tag{20b}$$

$$S_{21} = \beta_2(1-\Gamma_1\Gamma_{1p})/\beta_3 \tag{20c}$$

$$S_{22} = (\Gamma_{2p} - \Gamma_1\beta_1\beta_2)/\beta_3 \tag{20d}$$

$$\beta_1 = \frac{c_1}{c_2}\frac{\Gamma_{1a}-\Gamma_{1p}}{1-\Gamma_1\Gamma_{1a}} \tag{20e}$$

$$\beta_2 = \frac{c_2}{c_1}\frac{\Gamma_{2a}-\Gamma_{2p}}{1-\Gamma_2\Gamma_{2a}} \tag{20f}$$

$$\beta_3 = 1-\Gamma_1\Gamma_2\beta_1\beta_2. \tag{20g}$$

In the measurement process it is not necessary to change the direction of the two-port network under test. In three switch states, 16 power readings are obtained and all S_{ij}, i, $j=1,2$, are determined. The procedures for calibration and measurement are very simple.

IV. Calibration and Measurement of TSPNA

In order to find all the scattering parameters of an unknown three-port network, a TSPNA can be used. The TSPNA system is shown in Fig. 3. Compared with the DSPNA, the TSPNA has just three branches, each of which has the same structure as the two branches of the DSPNA. The associated devices have the same function as in the DSPNA. The difference is that the TSPNA has a three-branch divider.

The unknown three-port network is connected at reference planes 1, 2, and 3. Γ_1, Γ_2, Γ_3, c_1, c_2, and c_3 are system constants with the same definitions as in Section III. The isolators are ideal, so Γ_1, Γ_2, and Γ_3 do not change when switches change over, and the ratios of c_2 and c_3 to c_1 remain constant when the properties of the network under test change. The calibration steps are as follows.

First, the measurement port of sp_1 is directly connected to that of sp_2, so sp_1 and sp_2 constitute a DSPNA. According to the calibration method of the DSPNA as described in Section III, Γ_1, Γ_2, and c_2/c_1 can be obtained.

Then, let the measurement port of sp_1 be directly connected to that of sp_3; as described above, Γ_3 and c_3/c_1 can be obtained. Because the isolators used in the TSPNA are ideal, the TSPNA has only five system constants. By directly connecting sp_1 to sp_2 and sp_3, respectively, and measuring under five switch states, 20 power readings are obtained and the five system constants can be found from (13)–(19). The calibration procedure is very simple, and no standard three-port network is needed to calibrate the system constants.

The measurement steps of three-port networks are as follows. After successively calibrating the three six-ports sp_1, sp_2, sp_3, and finding the five system constants, we connect the unknown three-port network to the TSPNA, e.g. connect the three ports of the unknown network to the associated three measurement ports of the TSPNA. First, we measure the three-port network according to the ROM method.

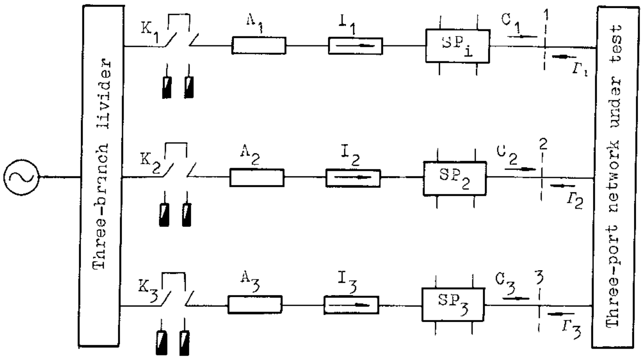

Fig. 3. A scheme of TSPNA.

1) Let K_2 and K_3 be closed, and K_1 open. The first port of the three-port network under test equals the connection of a load with the reflection coefficient of Γ_1 (which is composed of attenuator A_1, isolator I_1, and six-port sp_1). The three-port network and the load (Γ_1) constitute a two-port network [1] which is measured by a DSPNA composed of six-ports sp_2 and sp_3. From (20) $S_{22}^{(1)}, \cdots, S_{33}^{(1)}$ can be obtained. The subscripts of each quantity in (20) should be suitably substituted.

2) Let K_1 and K_3 be closed, and K_2 open. The second port of the three-port network is equal to the connection of a load with the reflection coefficient of Γ_2 (which is composed of A_2, I_2 and sp_2) and it is measured by a DSPNA composed of sp_1 and sp_3. Then $S_{11}^{(2)}, \cdots, S_{33}^{(3)}$ can be obtained.

3) Similarly, let K_1 and K_2 be closed, and K_3 open. $S_{11}^{(3)}, \cdots, S_{22}^{(3)}$ can be found by sp_1 and sp_2.

So far 12 scattering parameters $S_{ij}^{(k)}$, i, j, $k=1,2,3$ and i, $j \neq k$, have been obtained. From (8) the diagonal elements S_{11}, S_{22}, and S_{33} can be found. Equation (10) gives three linear equations for the nondiagonal elements S_{ij}, $i \neq j$, as follows:

$$S_{21}^{(3)}S_{12} + S_{12}^{(3)}S_{21} = S_{12}^{(3)}S_{21}^{(3)} + S_{12}S_{21} - (S_{11}^{(3)} - S_{11})(S_{22}^{(3)} - S_{22}) \tag{21a}$$

$$S_{31}^{(2)}S_{13} + S_{13}^{(2)}S_{31} = S_{13}^{(2)}S_{31}^{(2)} + S_{13}S_{31} - (S_{33}^{(2)} - S_{33})(S_{11}^{(2)} - S_{11}) \tag{21b}$$

$$S_{32}^{(1)}S_{23} + S_{23}^{(1)}S_{32} = S_{23}^{(1)}S_{32}^{(1)} + S_{23}S_{32} - (S_{22}^{(1)} - S_{22})(S_{33}^{(1)} - S_{33}). \tag{21c}$$

In order to find the solutions to (21) without any ambiguity, we must measure once more to find another three linear equations for the nondiagonal elements.

4) Let K_1, K_2, and K_3 all be closed. Then sp_1, sp_2, and sp_3 constitute a complete TSPNA. Each six-port yields a reflection coefficient, denoted by the subscript a; that is, Γ_{1a} is measured by sp_1, Γ_{2a} by sp_2, and Γ_{3a} by sp_3. Set

$$\Gamma_{ia} = b_i/a_i, \qquad i=1,2,3. \tag{22}$$

Substituting (22) into (1), we have

$$(S_{11}-\Gamma_{1a})a_1+S_{12}a_2+S_{13}a_3=0 \quad (23a)$$
$$S_{21}a_1+(S_{22}-\Gamma_{2a})a_2+S_{23}a_3=0 \quad (23b)$$
$$S_{31}a_1+S_{32}a_2+(S_{33}-\Gamma_{3a})a_3=0 \quad (23c)$$

where a_1, a_2, and a_3 are the results of c_1, c_2, and c_3 together and can be found with the superposition theorem.

Using a signal flow graph for the three-port network under test at the connected three references of the TSPNA, a_1, a_2, and a_3 can be easily found from the signal flow graph to be

$$\begin{aligned}a_1=&c_1(1-(S_{22}\Gamma_2+S_{33}\Gamma_3+\Gamma_2\Gamma_3S_{23}S_{32})+\Gamma_2\Gamma_3S_{22}S_{33})\\&+c_2(S_{12}\Gamma_1(1-S_{33}\Gamma_3)+\Gamma_1\Gamma_3S_{13}S_{32})\\&+c_3(S_{13}\Gamma_1(1-S_{22}\Gamma_2)+\Gamma_1\Gamma_2S_{12}S_{23}) \quad (24a)\end{aligned}$$
$$\begin{aligned}a_2=&c_1(S_{21}\Gamma_2(1-S_{33}\Gamma_3)+\Gamma_2\Gamma_3S_{23}S_{31})\\&+c_2(1-(S_{11}\Gamma_1+S_{33}\Gamma_3+\Gamma_1\Gamma_3S_{13}S_{31})+\Gamma_1\Gamma_3S_{11}S_{33})\\&+c_3(S_{23}\Gamma_2(1-S_{11}\Gamma_1)+\Gamma_1\Gamma_2S_{13}S_{21}) \quad (24b)\end{aligned}$$
$$\begin{aligned}a_3=&c_1(S_{31}\Gamma_3(1-S_{22}\Gamma_2)+\Gamma_2\Gamma_3S_{21}S_{32})\\&+c_2(S_{32}\Gamma_3(1-S_{11}\Gamma_1)+\Gamma_1\Gamma_3S_{12}S_{31})\\&+c_3(1-(S_{11}\Gamma_1+S_{22}\Gamma_2+\Gamma_1\Gamma_2S_{12}S_{21})+\Gamma_1\Gamma_2S_{11}S_{22}) \quad (24c)\end{aligned}$$

where the graph determinant is neglected, because it will be eliminated in the following operation. Substituting (24) into (23), we find another three linear equations for the nondiagonal elements:

$$(1-S_{33}\Gamma_3)\Gamma_2S_{21}^{(3)}S_{12}+(1-S_{22}\Gamma_2)\Gamma_3S_{31}^{(2)}S_{13}=R_1 \quad (25a)$$
$$(1-S_{33}\Gamma_3)\Gamma_1S_{12}^{(3)}S_{21}+(1-S_{11}\Gamma_1)\Gamma_3S_{32}^{(1)}S_{23}=R_2 \quad (25b)$$
$$(1-S_{22}\Gamma_2)\Gamma_1S_{13}^{(2)}S_{31}+(1-S_{11}\Gamma_1)\Gamma_2S_{23}^{(1)}S_{32}=R_3 \quad (25c)$$

TABLE II

Number of ports	3	4	5
Number of connections			
Two-port network analyzer	3	6	10
ROM	1	4	10

where

$$\begin{aligned}R_1=&(\Gamma_{1a}-S_{11})(1-S_{22}\Gamma_2-S_{33}\Gamma_3\\&+\Gamma_2\Gamma_3(S_{22}S_{33}-S_{23}S_{32}))\\&-(1-\Gamma_{1a}\Gamma_1)(1-S_{33}\Gamma_3)S_{12}^{(3)}c_2/c_1\\&-(1-\Gamma_{1a}\Gamma_1)(1-S_{22}\Gamma_2)S_{13}^{(2)}c_3/c_1 \quad (26a)\end{aligned}$$
$$\begin{aligned}R_2=&(\Gamma_{2a}-S_{22})(1-S_{11}\Gamma_1-S_{33}\Gamma_3\\&+\Gamma_1\Gamma_3(S_{11}S_{33}-S_{13}S_{31}))\\&-(1-\Gamma_{2a}\Gamma_2)(1-S_{33}\Gamma_3)S_{21}^{(3)}c_1/c_2\\&-(1-\Gamma_{2a}\Gamma_2)(1-S_{11}\Gamma_1)S_{23}^{(1)}c_3/c_2 \quad (26b)\end{aligned}$$
$$\begin{aligned}R_3=&(\Gamma_{3a}-S_{33})(1-S_{11}\Gamma_1-S_{22}\Gamma_2\\&+\Gamma_1\Gamma_2(S_{11}S_{22}-S_{12}S_{21}))\\&-(1-\Gamma_{3a}\Gamma_3)(1-S_{22}\Gamma_2)S_{31}^{(2)}c_1/c_3\\&-(1-\Gamma_{3a}\Gamma_3)(1-S_{11}\Gamma_1)S_{32}^{(1)}c_2/c_3. \quad (26c)\end{aligned}$$

Combining (21) and (25) and writing them in matrix form, we have

$$MS^c=N \quad (27a)$$
$$S^c=(S_{12},S_{13},S_{21},S_{23},S_{31},S_{32})^t=(S_1,S_2,S_3,S_4,S_5,S_6)^t \quad (27b)$$
$$N=(P_1,P_2,P_3,R_1,R_2,R_3)^t=(N_1,N_2,N_3,N_4,N_5,N_6)^t \quad (27c)$$
$$P_1=S_{12}^{(3)}S_{21}^{(3)}+S_{12}S_{21}-(S_{11}^{(3)}-S_{11})(S_{22}^{(3)}-S_{22}) \quad (27d)$$
$$P_2=S_{13}^{(2)}S_{31}^{(2)}+S_{13}S_{31}-(S_{11}^{(2)}-S_{11})(S_{33}^{(2)}-S_{33}) \quad (27e)$$
$$P_3=S_{23}^{(1)}S_{32}^{(1)}+S_{23}S_{32}-(S_{22}^{(1)}-S_{22})(S_{33}^{(1)}-S_{33}) \quad (27f)$$

$$M=\begin{bmatrix} S_{21}^{(3)} & 0 & S_{12}^{(3)} & 0 & 0 & 0\\ 0 & S_{31}^{(2)} & 0 & 0 & S_{13}^{(2)} & 0\\ 0 & 0 & 0 & S_{32}^{(1)} & 0 & S_{23}^{(1)}\\ (1-S_{33}\Gamma_3)\Gamma_2S_{21}^{(3)} & (1-S_{22}\Gamma_2)\Gamma_3S_{31}^{(2)} & 0 & 0 & 0 & 0\\ 0 & 0 & (1-S_{33}\Gamma_3)\Gamma_1S_{12}^{(3)} & (1-S_{11}\Gamma_1)\Gamma_3S_{32}^{(1)} & 0 & 0\\ 0 & 0 & 0 & 0 & (1-S_{22}\Gamma_2)\Gamma_1S_{13}^{(2)} & (1-S_{11}\Gamma_1)\Gamma_2S_{23}^{(1)}\end{bmatrix} \quad (27g)$$

Using Cramer's rule from (27) we have

$$S_l=\sum_{h=1}^{6}N_hM_{hl}/\det M, \qquad l=1,2,\cdots,6 \quad (28)$$

where M_{hl} is an algebraic complementary minor of the element of the hth row and lth column of the matrix M. The relationship between S_{ij} and S_l is given by (27b). So all the scattering parameters of the three-port network under test have been obtained.

At first glance, it seems that the measurement of the three-port network requires four steps under ten switch states as above. In reality this is not the case. There are only three switches in the TSPNA, which can be expressed by a three-figure numeral in the binary system: 0 denotes the switch open, 1 denotes the switch closed. Except for 000, a three-figure numeral in the binary system expresses seven numerals at most; that is, all the scattering parameters of the three-port network can be obtained by measuring it under the seven switch states from the 48 power readings obtained.

In fact, the TSPNA is a universal six-port network analyzer (USPNA). It obviously has the functions of the SSPNA and the DSPNA and can measure a variety of microwave multiport networks. For example, in measuring four-port networks, only a load with an accurately known reflection coefficient of Γ_k is needed, but the Γ_k is not required to be a special value; thus this method is very convenient. To measure an N-port network, $(n-3)$ loads with accurately known reflection coefficients are needed, where $n>3$. The number of necessary connections can be reduced by using a TSPNA when $n=3$, 4, as shown in Table II.

V. Error Analysis

The simplicity of the ROM method lies in the ideal isolators used in the system. It is difficult to realize ideal isolation. The S_{12} of the isolators does not always equal zero. The errors brought by nonideal isolation must be discussed.

The structure of the TSPNA is symmetrical, so we can discuss one branch as an example. The circuit between reference plane i and switch K_i can be equivalent to the one shown in Fig. 4, where S^{sp_i} denotes the scattering parameters of the two-port network which is composed of the six-port sp_i and its four power meters. S^i denotes the scattering parameters of a practical isolator. Z_s is the source impedance seen looking into the divider from switch K_i. Γ are the reflection coefficients seen looking into the switch K_i and the power divider D from isolator I_i. Switchover of the switch K_i will be the cause of changes in Γ and b_i/a_i. It is easy to find that b_i/a_i is

$$b_i/a_i = S_{11}^{sp_i} + I'CF \tag{29a}$$

where

$$F=\frac{1+AI}{1-ACI} \tag{29b}$$

$$A=\frac{\Gamma}{1-S_{22}^{i}\Gamma} \tag{29c}$$

$$C=\frac{S_{11}^{i}S_{22}^{sp_i}}{1-S_{11}^{i}S^{sp}} \tag{29d}$$

$$I=S_{12}^{i}S_{21}^{1}/S_{11}^{i} \tag{29e}$$

$$I'=S_{12}^{sp_i}S_{21}^{sp_i}/S_{11}^{sp_i}. \tag{29f}$$

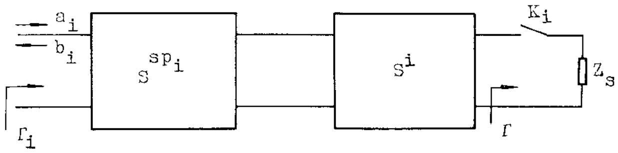

Fig. 4 The equivalent circuit between reference plane i and switch K_i.

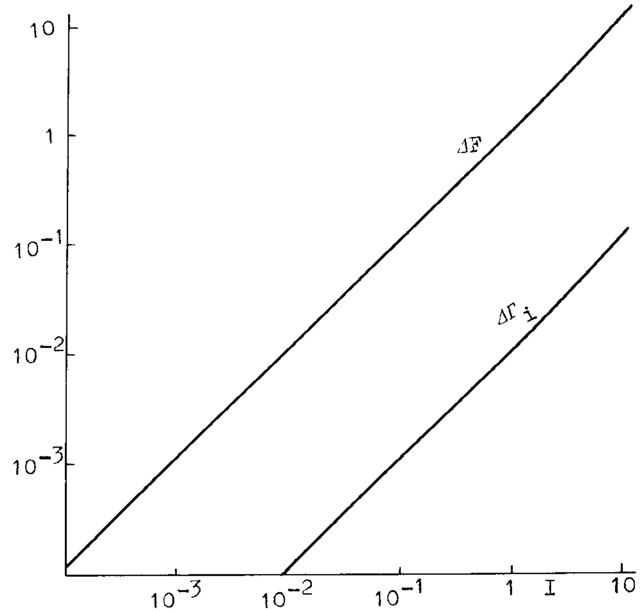

Fig. 5. The relationships between ΔF, ΔF_i, and I

A is a factor which describes the state of the switch; C is a factor which describes the matching situation between six-port sp_i and isolator I_i; I is the isolation factor of the isolator and I' is similarly the isolation factor of the six-port sp_i; and F is an error factor. For the ideal isolator $I=0$ and $F=1$ and we have

$$\Gamma_i=\frac{b_i}{a_i}=S_{11}^{sp_i}+I'C. \tag{30}$$

Equation (30) does not include the factor A, and Γ_i is always a constant, regardless of the state of switch K_i. For nonideal isolator $I\neq 0$ and $F\neq 1$, switchover of switch K_i will affect the value of Γ_i. The more F deviates from unity, the greater the error. When K_i is closed, $\Gamma=0$; when K_i is open, $\Gamma=1$. The change in the error factor between the two switch states is

$$F=F_{\Gamma=1}-F_{\Gamma=0}=\frac{AI(1+C)}{1-ACI} \tag{31}$$

where $A=A(\Gamma=1)$. From (29) the error of Γ_i is

$$\Delta\Gamma_i=I'CF. \tag{32}$$

Thus the measuring errors in the scattering parameters caused by nonideal isolation can be calculated.

Usually $A(\Gamma)\approx 1$, $C\approx 0$, and $\Delta F\approx I$, so that $\Delta\Gamma_i\approx II'C$. It is evident that there are three causes which affect $\Delta\Gamma_i$: isolation factors of the isolator and of the six-port, and the matching situation C between the six-port and the isolator. The more I, I', and C approach zero, the smaller $\Delta\Gamma_i$

becomes. The relationships between ΔF, $\Delta\Gamma_i$, and I are given in Fig. 5, where we have assumed that $S_{11}^{i} = S_{22}^{i} = 0.1$, $S_{11}^{\mathrm{sp}_i} = S_{22}^{\mathrm{sp}_i} = 0.1$, and $I' = 1$.

On the other hand, the six-port network has some isolation effect like the isolator, and it must be considered in the design of the TSPNA. The changes of C_2/C_1 and C_3/C_1 caused by nonideal isolation can be analyzed by the same method used above, and should produce the same conclusions.

VI. Conclusions

To realize the universal six-port network analyzer (USPNA) we have first discussed the general theory of measuring multiport networks, and suggested the ROM method for measuring microwave networks. All formulas for calibrating and measuring TSPNA given above are linear ones without any ambiguity, avoiding the inverse operation required by renormalization transforms. The analyses show that the TSPNA is a USPNA; it can directly measure one-, two-, and three-port networks. It can also measure multiport networks with $(n-3)$ known loads by the ROM method, reducing the number of connections. TSPNA should be the goal sought in designing six-port network analyzers.

The ROM method greatly simplifies the procedures of measuring multiport networks. The calibration and measurement of the system described above are very simple and do not require any standard three-port network. The simplicity of the ROM method lies in the ideal isolators used in the system. In practice, we cannot realize ideal isolation, and nonideal isolation will incur some errors. The error analysis shows that the isolation property of the isolator can be characterized by an isolation factor I, and that we should realize as accurately as possible ideal isolation through elaborate design.

Appendix

From (3) we have

$$\Gamma_k S_{ik} S_{kj} = \left(S_{ij}^{(k)} - S_{ij}\right)(1 - \Gamma_k S_{kk}). \tag{A1}$$

By retaining the constant k and exchanging subscripts i and j of the scattering parameters in (A1), we get another equation:

$$\Gamma_k S_{jk} S_{ki} = \left(S_{ji}^{(k)} - S_{ji}\right)(1 - \Gamma_k S_{kk}). \tag{A2}$$

Multiplying both sides of (A1) and (A2), respectively, we have

$$\begin{aligned}\Gamma_k^2 S_{ik} S_{ki} \cdot S_{jk} S_{kj} &= (1 - \Gamma_k S_{kk})^2 \left(S_{ij}^{(k)} - S_{ij}\right)\left(S_{ji}^{(k)} - S_{ji}\right)\\ &= (1 - \Gamma_k S_{kk})^2 \left(S_{ij}^{(k)} S_{ji}^{(k)} + S_{ij} S_{ji} - S_{ij}^{(k)} S_{ji} - S_{ji}^{(k)} S_{ij}\right).\end{aligned}$$

Dividing the above equation by $(1 - \Gamma_k S_{kk})^2$ yields

$$\frac{\Gamma_k S_{ik} S_{ki}}{1 - \Gamma_k S_{kk}} \cdot \frac{\Gamma_k S_{ik} S_{kj}}{1 - \Gamma_k S_{kk}} = S_{ij}^{(k)} S_{ji}^{(k)} + S_{ij} S_{ji} - S_{ij}^{(k)} S_{ji} - S_{ji}^{(k)} S_{ij}. \tag{A3}$$

Because

$$\frac{\Gamma_k S_{ik} S_{ki}}{1 - \Gamma_k S_{kk}} = S_{ii}^{(k)} - S_{ii}$$

(A3) becomes

$$\left(S_{ii}^{(k)} - S_{ii}\right)\left(S_{jj}^{(k)} - S_{jj}\right) = S_{ij}^{(k)} S_{ji}^{(k)} + S_{ij} S_{ji} - S_{ij}^{(k)} S_{ji} - S_{ji}^{(k)} S_{ij}$$

that is,

$$S_{ij}^{(k)} S_{ji} + S_{ji}^{(k)} S_{ij} = S_{ij}^{(k)} S_{ji}^{(k)} + S_{ij} S_{ji} - \left(S_{ii}^{(k)} - S_{ii}\right)\left(S_{jj}^{(k)} - S_{jj}\right).$$

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